Revenue Capacity Efficiency of Pear Trees and Its Decomposition

Chenggang Wang, Rolf Färe, and Clark F. Seavert

Department of Agricultural & Resource Economics, Oregon State University, Ballard Extension Hall, Corvallis, OR 97331

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ABSTRACT. In this paper we analyze the sources of variation in revenue per unit of trunk cross-sectional area (TCA) across a 0.87-ha block of 272 pear (*Pyrus communis* L.) trees in 2003. Revenue capacity efficiency associated with TCA provides an overall measure of nutrient deficiency and revenue inefficiency caused by environmental constraints in the fruit production process. Data envelopment analysis (DEA) is adopted to estimate revenue capacity efficiency and its components. The deficiencies of macro- and micronutrients are measured and optimal nutrient levels computed for each individual tree. These measures are aggregated for comparing between grids and between rootstocks.

Yield efficiency, fruit weight per unit of trunk cross-sectional area (TCA), has long been used as a measure of the fruit tree's productivity. The argument for the usefulness of this measure is that TCA has been shown to be linearly correlated with total aboveground weight of the tree, and thus fruit weight per unit of TCA reflects the efficiency of the tree's potential bearing surface to produce fruit (Westwood and Roberts, 1970). The value-based measure of yield efficiency-economic value of fruit per unit of TCA-takes into account the quality and market value of fruit and thus measures the tree's economic performance. Accounting for variation in this measure across a commercial orchard can help to identify potential problems of individual trees, provide sound management recommendations, and enhance orchard profitability. Unfortunately, this commonly observed variation has not been seriously studied and oftentimes is attributed to unexplained statistical disturbances in a regression of tree revenue on tree TCA (e.g., Seavert, 2002).

Fruit production is the use of essential nutrients and sunlight to manufacture fruit through the process of photosynthesis. Generally, deficiency of essential nutrients in the soil and productive inefficiency are the two potential sources of relatively low revenue per unit of TCA.

Sixteen nutrients are considered essential to plant growth; they are carbon, hydrogen, oxygen, nitrogen, phosphorous, potassium, calcium, magnesium, sulfur, iron, manganese, zinc, copper, boron, molybdenum, and chlorine. Each of the essential nutrients plays its unique, indispensable role in plant growth (Fageria, 1992). According to von Liebig's Law of the Minimum, crop yields are in direct relation with the minimum of one or more nutrients (Havlin et al., 1999). To the extent that TCA is a proxy of the tree's potential bearing surface and thus capacity of photosynthesis, revenue per unit of TCA is essentially a measure of revenue capacity utilization. Deficiency of any of the essential nutrients in the soil can limit the realization of revenue capacity, resulting in relatively low revenue per unit of TCA.

A tree with sufficient essential nutrients in the soil may as well generate relatively low revenue in view of its potential bearing surface, owing to the constraint of environmental factors in the fruit production process. This is the so-called revenue inefficiency (a formal definition will be given in the next section). The potential environmental constraints include soil factors, climate factors, agronomic factors, biological factors and so forth. A low level of organic matter, for example, can reduce the soil's water holding capacity and exchangeable K, Ca, and Mg, and thus limit yield and revenue (Baligar et al., 2001). As another example, infections of diseases can reduce tree yield and therefore tree revenue.

In this paper we develop a fruit production model to analyze these two sources of variation in tree revenue per unit of TCA for a block of 272 pear trees.

Materials and Methods

DATA COLLECTION. The Hardy block at the Oregon State Univ. Mid-Columbia Agricultural Research and Extension Center (OSU-MCAREC) was established in 1974 with the research objective of comparing four rootstocks-'Old Home × Farmingdale 69' ('OH×F 69'), 'OH×F 97', 'OH×F 217', and 'OH×F 333' - which were grafted to the d'Anjou cultivar for commercial application. Figure 1 is the elevation map of the Hardy block with tree location denoted. The elevation of the block ranges from 150 to 166 m above sea level with a northwest-facing slope. The block is 0.87 ha in size with 10 rows of trees planted east to west. The row length varies from 29 to 56 trees. In six rows of the ten, the four rootstocks were planted in consecutive sets of four so that every fourth rootstock was the same. During the past several years this block has been farmed for commercial pear production with no modification in production practices for any of the four rootstocks. The average yield in the last 4 years was ≈99 bins/ha, which is $\approx 25\%$ above the county average production.

To collect soil nutrient data, the block was divided into 14 grids (Fig. 1). In Nov. 2002, a soil sample was collected in the center of each grid. Each sample consists of about eight to 10 cores of soil collected at the 50-cm depth. Eleven essential mineral nutrients for plant growth [including two forms of nitrogen, nitrate nitrogen (NO₃-N), and ammonium nitrogen (NH₄-N)] were analyzed and results were reported for each grid. These nutrients are listed in Table 1 as inputs of our model together with trunk cross-sectional area (TCA), which was measured for each tree in the same year. Among the 16 essential nutrients, C, O, H, Mo, and Cl are missing in our dataset. Carbon, O, and H are derived from atmospheric CO₂ and soil water. Carbon dioxide and sunlight are unlimitedly available and thus can be neglected

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Elevation map and tree location of the Hardy block

Fig. 1. The Hardy block is 0.87 ha in size and has a northwest-facing slope. In the six rows of trees, the four rootstocks were planted in consecutive sets of four so that every fourth rootstock was the same. The block was divided into 14 grids, and a soil sample was collected for each grid in Nov. 2002. A packout was obtained for each of the 272 pear trees in Sept. 2003.

from our analysis. However, the same argument does not apply to water, Mo, and Cl. Throughout this study we assume that they have been sufficient in the entire production period.

In order to observe revenue variability on an individual tree basis, this block was harvested and a packout was derived for each of the 272 trees in Sept. 2003. First, fruit of each tree was gathered into individual bins. Then, each bin was analyzed for weight of total fruit, weight of culls, and distribution of fruit size by the OSU–MCAREC packing line, resulting in a dataset that contains the yields of 21 categories of pears for each individual tree. In order to minimize the number of outputs, categories with the same market price were further combined into one, leaving us with 12 categories of pears as outputs of our model (Table 1). Finally, the commercial pear price was applied to each category, yielding a value for each tree.

Figure 2 is the scatter plot of tree revenue against TCA for the 272 trees. This plot indicates a wide variation in tree revenue per unit of TCA across this block. Tree M registered the highest revenue per unit of TCA, whereas a bigger tree like N generated much less revenue than tree M. Assume that ABCDE is the revenue–TCA profile when there are no deficient nutrients and revenue inefficiency whatsoever. The boundary point for tree N is Q, whose y-coordinate, equal to PQ, represents this tree's revenue capacity obtainable by eliminating deficiency of essential nutrients and revenue inefficiency completely. The ratio of tree N's revenue capacity to observed revenue, PQ/PN, provides a measure of the overall limiting effect of deficient nutrients and environmental constraints.

Next we decompose this measure into the components of nutrient deficiency and revenue inefficiency. We also address the problem of how to aggregate these measures for comparing between grids and between rootstocks, while still preserving the decomposition structure.

THEORETICAL MODEL. In our model, pear production is viewed as a process of transforming essential nutrients and sunlight into various categories of pears. Specifically, inputs include 11 essential nutrients and TCA, with sunlight and the other essential nutrients being assumed to be unlimitedly available; and outputs include 12 categories of pears sorted by size and grade (Table 1).

Some notation is required to interpret the model. Thus, denote inputs by $x = (x_1, ..., x_N) \in \mathfrak{R}^N_+$ and outputs by $y = (y_1, ..., y_M) \in \mathfrak{R}^M_+$. The *output possibilities set* is defined as

$$P(x) = \{y \in \mathfrak{R}^{M}_{+} : x \in \mathfrak{R}^{N}_{+}, x \text{ can produce } y\}$$
 Eq. [1]

This set consists of all the combinations of outputs that can be produced by input vector x, and thus describes our "tree" technology.

The output distance function $D_o: \Re^M_+ \times \Re^N_+ \to \Re_+ \cup \{+\infty\}$, is defined as

$$D_o(x, y) = \inf_{\theta} \{\theta > 0 : y/\theta \in P(x)\}.$$
 Eq. [2]

This function inherits the properties of the output possibilities set. Under certain conditions (Färe and Primont, 1995), the latter can be completely described by the former, i.e.,

$$P(x) = \{y : D_o(x, y) \le 1\}.$$
 Eq. [3]

The *Farrell output-oriented technical efficiency index* is defined as the reciprocal of the output distance function, i.e.

$$TE(x, y) = 1/D_o(x, y)$$
. Eq. [4]

	Unit of					Price
Variable	measure	Minimum	Mean	Maximum	SD	(\$/kg)
Input						
NO ₃ -N	mg∙kg⁻¹	1.00	2.77	5.00	0.98	
NH ₄ -N	mg∙kg⁻¹	2.00	2.90	6.00	1.03	
Р	mg∙kg⁻¹	49.00	72.66	104.00	13.24	
Κ	mg∙kg⁻¹	233.00	303.03	369.00	32.49	
Ca	meq/100 g	4.90	5.79	6.80	0.51	
Mg	meq/100 g	1.50	1.82	2.30	0.22	
S	mg∙kg⁻¹	5.40	8.05	10.20	1.53	
В	mg·kg ^{−1}	0.30	0.42	0.60	0.07	
Zn	mg∙kg ⁻¹	7.20	12.05	20.00	3.12	
Mn	mg∙kg ⁻¹	4.00	5.28	11.00	1.76	
Cu	mg∙kg ⁻¹	6.70	17.09	34.00	7.60	
Fe	mg∙kg⁻¹	77.00	89.89	110.00	10.11	
TCA ^z	cm ²	134.42	472.11	786.25	114.20	
Output						
Culls ^y	kg	7.23	35.86	80.31	10.61	0.018
US#1(<90)x	kg	6.53	100.57	269.64	41.08	0.265
US#1(100,110)x	kg	11.55	43.24	103.87	14.86	0.258
US#1(120)x	kg	1.92	9.70	25.54	4.51	0.254
US#1(135)x	kg	0.77	5.40	14.74	3.00	0.220
US#1(150)x	kg	0.18	3.83	23.77	3.19	0.121
Fancy(<60) ^x	kg	0.00	4.04	52.13	6.69	0.106
Fancy(80) ^x	kg	0.90	16.69	32.98	6.82	0.205
Fancy(70,90,100)x	kg	5.36	39.66	78.45	12.01	0.179
Fancy(110,120)x	kg	3.19	12.47	33.12	5.07	0.159
Fancy(135) ^x	kg	0.36	2.56	7.00	1.42	0.115
Fancy(150) ^x	kg	0.08	1.82	11.28	1.52	0.104

^zTCA = trunk cross-sectional area.

y"Culls" are pears that are not sold on the fresh market but may have value in the processing market.

*"US#1" consists of the U.S. No 1 grade pears, while "Fancy" consists of the U.S. No 2 grade pears. Number in parentheses refers to fruit size. For example, "US#1(<90)" represents the U.S. No 1 grade pears that have less than 90 pears in a standard 20-kg box, and "Fancy(70,90,100)" represents the U.S. No 2 grade pears that have 70, 90, or 100 pears in a standard 20-kg box. The other outputs are analogously defined.

Note that for this index efficiency is indicated by value 1 and inefficiency by values larger than 1. That is to say, a higher score is not better. This is also true for all the other indexes in our model.

We illustrate these definitions with a two-output model in Fig. 3. Assume that the two outputs are pears of high quality and low quality, measured by y_H and y_L , respectively. The output possibilities set P(x), the area OSS', consists of all outputs producible from input vector x. The output distance function for a tree represented by point A is the ratio OA/OB. It can be seen that the value of the distance function is equal to 1 for all the points on the frontier SS', <1 for those in the interior of P(x), and >1 for those outside SS'. In this case P(x) is the set of points with the value of the distance function ≤ 1 .

Trees represented by points on the production frontier SS' are technically efficient in the sense that the outputs can not be proportionally increased any more. A tree represented by point A is not technically efficient in that a radial expansion in both outputs is attainable with the same inputs and technology. The Farrell technical efficiency index provides a measure of how far outputs can be proportionally increased with the given inputs and technology.

This measure does not take into account the market values of outputs. Suppose, without loss of generality, that high-quality pears have a higher market price than low-quality pears. Let the slope of line LL' in Fig. 3 be the ratio of the two prices, P_H/P_L . All points on LL' generate the same level of revenue, and a line farther from the origin represents a higher level of revenue. Now the optimal method of production is represented by B' instead of B. Although both points are technically efficient, the production at B' returns higher revenue than that at B. In fact, LL' represents the highest revenue associated with P(x) given the prices p_H and p_L . An efficiency measure that takes into account the market values of outputs is the ratio OC/OA. To formalize this, define the maximal revenue as

$$R(x, p) = \max\{py : y \in P(x)\},$$
 Eq. [5]

where *p* is the vector of output prices, and $py = \sum_{j=1}^{M} p_j y_j$. The *Farrell* revenue efficiency index is defined as:

$$RE(x, y, p) = R(x, p)/py$$
 Eq. [6]

where R(x, p) is the maximal revenue and py is the observed revenue.

It follows that the technical efficiency measure is a component of the revenue efficiency measure. The residual component is the so-called *Farrell output-oriented allocative efficiency index*:



Fig. 2. Scatter plot of tree revenue vs. trunk cross-sectional area (TCA) for the 272 pear trees denoted in Fig. 1. Tree M achieved the highest revenue per unit of TCA. ABCED is assumed to be the revenue-TCA profile when there are no nutrient deficiency and revenue inefficiency whatsoever. PN measures tree N's observed revenue, while PQ measures its revenue capacity obtainable by eliminating deficiency of essential nutrients and revenue inefficiency completely. PO/PN measures the overall limiting effect of deficient nutrients and environmental constraints.



Fig. 3. Decomposition of revenue efficiency in a two-output pear production model. Point A represents tree A's yields of high- and low-quality pears (denoted by y_H and y_L with input vector x. P(x), the area OSS', is the output possibilities set. LL' is the iso-revenue line, whose slope is equal to the price ratio of high- and low-quality pears, p_H/p_L . OB/OA is tree A's output-oriented technical efficiency index (TE), OC/OA is the revenue efficiency index (RE), and OC/OB is the allocative efficiency index (AE). The decomposition expression $RE = TE \times AE$ translates into $\frac{OC}{OA} = \frac{OB}{OA} \times \frac{OC}{OB}$ in the case of tree A.

$$AE(x, y, p) = \frac{RE(x, y, p)}{TE(x, y)}.$$
 Eq. [7]

Rearranging Eq. [7] yields a decomposition of the Farrell revenue efficiency measure:

In the case of tree A in Fig. 3, this decomposition translates into $\frac{OC}{OL} = \frac{OB}{OL} \times \frac{OC}{OB}$

 $OA ^{-}OA ^{-}OB$

To further expand this decomposition we introduce the concept of capacity. Johansen (1987) defines a plant's capacity as "...the maximum amount that can be produced per unit of time with the existing plant and equipment provided that the availability of variable factors is not restricted." In our framework, a pear tree's revenue capacity can be defined as the maximum revenue obtainable provided the availability of variable nutrients is not restricted.

Following Färe et al. (1989), let the input vector be partitioned into two subvectors, i.e., $x = (x_{e})$ x_{v}), where f refers to the fixed factors and v refers to the variable factors. In our particular case, the variable factors include nutrients that can be easily adjusted by the orchard manager, whereas the fixed factors include TCA and nutrients that are costly or impossible to adjust. When the availability of variable factors is not restricted, the output possibilities set becomes

$$\widetilde{P}(x_f) = \{ y \in P(x_f, x_v) : x_v \ge 0 \}.$$
 Eq. [9]

Thus, we define the pear tree's revenue capacity as

$$\hat{R}(x_f, p) = \max_{y} \{ py : y \in \widetilde{P}(x_f) \} = \max_{y, x_v} \{ py : y \in P(x_f, x_v), x_v \ge 0 \}$$

Eq. [10]

It follows that $\hat{R}(x_p, p) \ge R(x, p)$ and the revenue capacity can be realized only if all of the variable nutrients are at the optimal levels. Thus, the ratio of the revenue capacity Eq. [10] to the maximal revenue Eq. [5] provides a measure of the limiting effect of variable nutrients, or a measure of how responsive the maximal revenue is to the adjustment of variable nutrients. We define the nutrient responsiveness index as

$$VR(x_f, x_v, p) = \frac{\hat{R}(x_f, p)}{R(x_f, x_v, p)}.$$
 Eq. [11]

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Notice that we can investigate the constraint of certain group of nutrients in question-for example, micronutrients or macronutrients—by varying the structure of variable and fixed input subvectors.

Next, we define the revenue capacity efficiency index as

$$RCE(x_f, y, p) = \hat{R}(x_f, p) / py, \qquad \text{Eq. [12]}$$

where $\hat{R}(x_p, p)$ is the revenue capacity and py is the observed revenue. In particular, when TCA is defined as the only fixed input factor, this index reduces to a measure of how far a tree's revenue per unit TCA falls short of the maximal revenue per unit TCA attainable. Returning to the scatter plot of tree revenue against tree TCA (Fig. 2), this measure for tree N is just its revenue capacity divided by observed revenue, PQ/PN, which provides an overall measure of nutrient deficiency and revenue inefficiency caused by environmental factors.



Fig. 4. Decomposition of revenue capacity efficiency in a two-output pear production model. Point *A* represents tree *A*'s yields of high- and low-quality pears (denoted by y_H and y_L) with input vector *x*. $\tilde{P}(x)$, the area *OSS'*, is the output possibilities set associated with input vector *x*, while $\tilde{P}(x)$, the area *OTT'*, is the output possibilities set when the availability of variable factors is not restricted. The slopes of *LL'* and *II'* are both equal to the price ratio of high- and low-quality pears, p_H/p_L . *OB/OA* is tree *A*'s output-oriented technical efficiency index (*TE*), *OC/OA* is the revenue efficiency index (*RE*), *OC/OB* is the allocative efficiency index (*AE*), and *OD/OC* and *OD/OA* are the nutrient responsiveness and revenue capacity efficiency indexes (*NR* and *RCE*) associated with the fixed factors x_p . The decomposition expression *RCE* = *TE* × *AE* × *NR* translates into $\frac{OD}{OA} = \frac{OB}{OA} \times \frac{OC}{OE} \times \frac{OD}{OC}$ in the case of tree *A*.

The revenue capacity efficiency index can be decomposed into the revenue efficiency index Eq. [6] and the nutrient responsiveness index Eq. [11], i.e.,

$$RCE(x_{\rho}, y, p) = RE(x_{\rho}, x_{v}, y, p) \times NR(x_{\rho}, x_{v}, p).$$
 Eq. [13]

In the case of tree A in Fig. 4, Eq. [13] translates into $\frac{OD}{OD} = \frac{OC}{OC} \times \frac{OD}{OD}$

 $OA = OA \cap OC$

Finally, substituting Eq. [8] into Eq. [13] yields

$$RCE(x_{p}, y, p) = TE(x, y) \times AE(x, y, p) \times NR(x_{p}, x_{y}, p).$$
 Eq. [14]

In the case of tree A in Fig. 4, Eq. [14] translates into $\frac{OD}{OD} = \frac{OB}{OD} \times \frac{OC}{OD} \times \frac{OD}{OD}$

The next problem is how to aggregate these indexes across the 14 grids, across the four rootstocks, and across the entire block, while preserving the multiplicative structure of these decompositions. Here we adopt a weighted geometric mean procedure suggested by Färe and Grosskopf (2004), and Färe and Zelenyuk (unpublished). A weighted geometric mean is necessary to preserve the multiplicative structure of the decompositions. The best weights are given by the revenue shares of individual trees. See Färe and Grosskopf (2004) for details.

ESTIMATION OF REVENUE CAPACITY EFFICIENCY AND ITS COM-PONENTS. From the foregoing discussion, the key to computing these indexes is to estimate the production frontier. One possible approach is to estimate a parametric frontier using the so-called stochastic frontier technique (Greene, 2003). The difficulty in

applying this approach to the present study is that the commonly adopted parametric models do not satisfy the von Liebig's Law of the Minimum. For example, none of the input factors of the Cobb–Douglas production function is *limititional* (Shephard and Färe, 1974), resulting in no real solution for the computation of revenue capacity. There is a strand of literature in agriculture economics that seeks the appropriate production functions that incorporate the Law of the Minimum (e.g., Paris, 1992). However, efficiency measurement becomes difficult in these models owing to the complex functional forms (Holloway and Paris, 2002).

The alternative, which is adopted here, is to employ an activity analysis approach or the data envelopment analysis (DEA) method. This approach estimates the production frontier without assuming any parametric functional form. Specifically, the production frontier is approximately formed by the best practical management units. This approximated frontier is called the best practical frontier. The measures defined earlier can be readily computed with the appropriate linear programs. For an exposition of this method, see Färe and Grosskopf (2000).

To illustrate how to compute the indexes defined earlier, assume there are k = 1,..., K pear trees that employ $x^k \in \mathfrak{R}^N_+$ inputs to produce $y^k \in \mathfrak{R}^M_+$ outputs. Under the assumptions of strong disposability of outputs and variable returns to scale [we tested for scale efficiency using the current data set, and the result led us to use the variable returns to scale model; for the definitions of returns to scale and disposability and how to test for scale efficiency, see Färe and Grosskopf (2000)], the output possibilities set can be estimated by solving the following linear program:

$$P(x) = \{(y_1, \dots, y_M) : \\ \sum_{k=1}^{K} z^k y_m^k \ge y_m, m = 1, \dots M, \\ \sum_{k=1}^{K} z^k x_n^k \le x_n, n = 1, \dots N, \\ \sum_{k=1}^{K} z^k = 1, z^k \ge 0, k = 1, \dots K\}.$$
 Eq. [15]

where the intensity variables z^k , k = 1,..., K, serve to form linear combinations of all observed trees' inputs and outputs.

For an arbitrary tree k', the Farrell output-oriented technical efficiency index is computed by

$$TE(x^{k'}, y^{k'}) = (D_o(x^{k'}, y^{k'}))^{-1} = \max \theta$$

s.t. $\sum_{k=1}^{K} z^k y_m^k \ge \theta y_m^{k'}, m = 1, \dots M,$
 $\sum_{k=1}^{K} z^k x_n^k \le x_n^{k'}, n = 1, \dots N,$
 $\sum_{k=1}^{K} z^k = 1, z^k \ge 0, k = 1, \dots K.$
Eq. [16]

The following program solves its maximum revenue:

$$R(x^{k'}, p) = \max_{y} \sum_{m=1}^{M} p_{m} y_{m}$$

s.t. $\sum_{k=1}^{K} z^{k} y_{m}^{k} \ge y_{m}, m = 1, \cdots M,$
 $\sum_{k=1}^{K} z^{k} x_{n}^{k} \le x_{n}^{k'}, n = 1, \cdots N,$
 $\sum_{k=1}^{K} z^{k} = 1, z^{k} \ge 0, k = 1, \cdots K.$
Eq. [17]

Table 2. Aggregate revenue capacity efficiency and its components for pear trees on the grid and block levels.

	RCEz	NRy	RE ^x	TEw	AE^{v}	RCE1 ^u	NR1 ^t	RCE2 ^s	NR2r
Grid 1	1.471	1.027	1.432	1.151	1.244	1.432	1.000	1.432	1.000
Grid 2	1.544	1.227	1.258	1.047	1.202	1.258	1.000	1.258	1.000
Grid 3	1.640	1.461	1.123	1.039	1.080	1.156	1.030	1.123	1.000
Grid 4	1.703	1.065	1.600	1.260	1.270	1.603	1.002	1.600	1.000
Grid 5	1.564	1.212	1.290	1.118	1.154	1.290	1.000	1.290	1.000
Grid 6	1.670	1.483	1.126	1.018	1.106	1.327	1.179	1.399	1.243
Grid 7	2.109	1.668	1.264	1.040	1.216	1.264	1.000	1.264	1.000
Grid 8	2.351	1.824	1.289	1.078	1.196	2.254	1.749	1.289	1.000
Grid 9	1.785	1.259	1.418	1.146	1.237	1.420	1.001	1.783	1.257
Grid 10	1.817	1.425	1.275	1.082	1.179	1.427	1.119	1.275	1.000
Grid 11	1.755	1.483	1.184	1.043	1.135	1.383	1.168	1.184	1.000
Grid 12	1.694	1.435	1.180	1.046	1.128	1.180	1.000	1.180	1.000
Grid 13	1.649	1.018	1.620	1.317	1.230	1.649	1.018	1.620	1.000
Grid 14	1.544	1.270	1.216	1.085	1.120	1.544	1.270	1.216	1.000
^q Hardy	1.662	1.282	1.296	1.105	1.173	1.385	1.068	1.329	1.025

 ${}^{2}RCE$ = revenue capacity efficiency index when trunk cross-sectional area (TCA) is defined as the only fixed input; $RCE = NR \times RE = NR \times AE \times TE.$

yNR = nutrient responsiveness index when TCA is defined as the only fixed input.

RE = Farrell revenue efficiency index.

 ^{w}TE = Farrell output-oriented technical efficiency index.

 ^{v}AE = Farrell output-oriented allocative efficiency index.

"RCE1 = revenue capacity efficiency index when macronutrients NO₃-N, NH₄-N, P, K, Ca, Mg, and S, are defined as variable inputs; $RCE1 = NR1 \times RE = NR1 \times AE \times TE$.

VR1 = nutrient responsiveness index when macronutrients NO₃-N, NH₄-N, P, K, Ca, Mg, and S, are defined as variable inputs.

RCE2 = revenue capacity efficiency index when micronutrients B, Zn, Mn, Cu, and Fe, are defined as variable inputs; $RCE2 = NR2 \times RE = NR2 \times AE \times TE$.

 $^{r}NR2$ = nutrient responsiveness index when micronutrients B, Zn, Mn, Cu, and Fe, are defined as variable inputs. ^qThe Hardy block as a whole.

Table 3. Revenue capacity efficiency when macronutrients NO₃-N, NH₄-N, P, K, Ca, Mg, and S, are defined as variable inputs and its components for pear trees in grid 8.

		1	1	1	0			
Row	Column	TCAz	Revenue	RCE1 ^y	NR1 ^x	REw	TE^{v}	AEu
G	49	355.09	22.22	1.452	1.452	1.000	1.000	1.000
G	50	459.64	21.28	1.983	1.899	1.044	1.000	1.044
G	51	429.9	10.48	3.854	1.817	2.121	1.282	1.654
G	52	365.8	15.78	2.117	1.504	1.408	1.000	1.408
G	53	305.9	13.06	2.053	2.053	1.000	1.000	1.000
G	54	356.16	11.46	2.825	1.457	1.939	1.270	1.527
Ι	49	457.22	4.97	8.466	1.893	4.473	2.708	1.652
Ι	50	479.2	19.62	2.209	1.951	1.132	1.000	1.132
Ι	51	469.37	18.64	2.295	1.925	1.192	1.000	1.192
Aggrega	ite measures			2.254	1.749	1.289	1.078	1.196

^zTCA = trunk cross-sectional area.

PRCE1 = revenue capacity efficiency index when macronutrients NO₃-N, NH₄-N, P, K, Ca, Mg, and S, are defined as variable inputs; $RCE1 = NR1 \times RE = NR1 \times AE \times TE$.

**NR1* = nutrient responsiveness index when macronutrients NO_3 -N, NH_4 -N, P, K, Ca, Mg, and S, are defined as variable inputs.

 $^{w}RE =$ Farrell revenue efficiency index.

 ^{v}TE = Farrell output-oriented technical efficiency index.

 ${}^{u}AE =$ Farrell output-oriented allocative efficiency index.

By definition, the Farrell revenue efficiency index for tree k' is

$$RE(x^{k'}, y^{k'}, p) = \frac{R(x^{k'}, p)}{py^{k'}}$$
Eq. [18]

and the Farrell output-oriented allocative efficiency index is

$$AE(x^{k'}, y^{k'}, p) = \frac{RE(x^{k'}, y^{k'}, p)}{TE(x^{k'}, y^{k'})}$$
Eq. [19]

Now let x be partitioned into two subvectors (x_{ρ}, x_{ν}) , where x_f denotes the F fixed factors and x_{ν} denote the V variable factors. The revenue capacity of tree k' can be computed by

$$\hat{R}(x_{f}^{k'}, p) = \max_{y} \sum_{m=1}^{M} p_{m} y_{m}$$

s.t. $\sum_{k=1}^{K} z^{k} y_{m}^{k} \ge y_{m}, m = 1, \dots M,$
 $\sum_{k=1}^{K} z^{k} x_{n}^{k} \le x_{n}^{k'}, n = 1, \dots F,$
 $\sum_{k=1}^{K} z^{k} = 1, z^{k} \ge 0, k = 1, \dots K.$
Eq. [20]

Table 4. Observed (OBS) and optimal (OPT) levels of variable macronutrients NO₃-N, NH₄-N, P, K, Ca, Mg, and S (defined as variable factors), for pear trees in grid 8.

		OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT
Row	Column	NO ₃ -N	NO ₃ -N	NH ₄ -N	NH ₄ -N	Р	Р	Κ	Κ	Ca	Ca	Mg	Mg	S	S
					(mg·kg ⁻¹)						_(meq/	100 g)		_(mg·	kg-1)_
G	49	2.0	2.4	3.0	2.8	86.0	62.1	369.0	284.7	5.9	5.7	1.8	1.9	7.0	8.1
G	50	2.0	3.0	3.0	2.7	86.0	59.0	369.0	258.6	5.9	5.3	1.8	1.7	7.0	7.4
G	51	2.0	3.0	3.0	2.7	86.0	59.1	369.0	259.3	5.9	5.3	1.8	1.7	7.0	7.4
G	52	2.0	2.5	3.0	2.8	86.0	61.7	369.0	281.1	5.9	5.7	1.8	1.8	7.0	8.0
G	53	2.0	2.5	3.0	2.9	86.0	62.3	369.0	292.8	5.9	5.9	1.8	1.9	7.0	8.3
G	54	2.0	2.5	3.0	2.8	86.0	62.0	369.0	284.1	5.9	5.7	1.8	1.9	7.0	8.1
Ι	49	2.0	3.0	3.0	2.7	86.0	59.1	369.0	258.9	5.9	5.3	1.8	1.7	7.0	7.4
Ι	50	2.0	3.0	3.0	2.7	86.0	59.1	369.0	258.9	5.9	5.3	1.8	1.7	7.0	7.4
Ι	51	2.0	3.0	3.0	2.7	86.0	59.1	369.0	258.9	5.9	5.3	1.8	1.7	7.0	7.4
Average	level	2.0	2.8	3.0	2.8	86.0	60.4	369.0	270.8	5.9	5.5	1.8	1.8	7.0	7.7

Table 5. Revenue capacity efficiency when micronutrients B, Zn, Mn, Cu, and Fe, are defined as variable inputs and its components for pear trees in grid 9.

Row	Column	TCAz	Revenue	PCE2v	NR2x	PEw	TEv	ΔEu
C	42	452.61	5 10	0.620	1.257	<u>6.267</u>	1.710	2.710
G	42	453.01	5.12	8.638	1.357	6.367	1./12	3./19
G	43	557.5	31.30	1.619	1.242	1.304	1.084	1.203
G	44	568.2	30.99	1.656	1.231	1.345	1.227	1.096
G	45	497.9	30.39	1.551	1.306	1.188	1.115	1.065
G	46	172.07	10.21	1.156	1.156	1.000	1.000	1.000
G	47	509.3	23.81	2.009	1.293	1.554	1.206	1.289
G	48	594.04	16.35	3.233	1.209	2.674	2.436	1.098
Ι	41	476.73	29.74	1.543	1.334	1.157	1.000	1.157
Ι	42	623.27	28.31	1.928	1.186	1.626	1.054	1.543
Ι	44	623.27	30.97	1.763	1.186	1.487	1.181	1.259
Ι	45	424.07	22.89	1.785	1.351	1.321	1.000	1.321
Ι	46	548.21	25.00	2.005	1.251	1.603	1.122	1.429
Ι	47	447.62	21.23	2.050	1.356	1.512	1.169	1.293
J	41	418.28	29.53	1.361	1.350	1.008	1.000	1.008
J	43	630.33	46.61	1.175	1.175	1.000	1.000	1.000
J	45	499.16	21.75	2.171	1.305	1.664	1.510	1.102
J	46	504.22	20.10	2.364	1.299	1.820	1.285	1.416
J	47	786.25	22.24	2.463	1.175	2.096	1.000	2.096
Aggregate med	asures			1.783	1.257	1.418	1.146	1.237

 $\overline{{}^{z}TCA}$ = trunk cross-sectional area.

RCE2 = revenue capacity efficiency index when micronutrients B, Zn, Mn, Cu, and Fe, are defined as variable inputs; RCE2 = NR2×RE = NR2×AE×TE.

^xNR2 = nutrient responsiveness index when micronutrients B, Zn, Mn, Cu, and Fe, are defined as variable inputs.

wRE = Farrell revenue efficiency index.

^vTE = Farrell output-oriented technical efficiency index.

^uAE = Farrell output-oriented allocative efficiency index.

Hence, the nutrient responsiveness index for tree k' is

$$NR(x_{f}^{k'}, p) = \frac{\hat{R}(x_{f}^{k'}, p)}{R(x^{k'}, p)}$$
Eq. [21]

At the same time, the optimal levels of variable nutrients can be computed by multiplying the vector of solutions for the intensity variables (z^{k}) by the vector of variable nutrients,

$$x_{\nu}^{k^*} = \sum_{k=1}^{K} z^{k^*} x_{\nu}^{k}, \nu = 1, \dots V.$$
 Eq. [22]

Finally, the revenue capacity efficiency index for tree k' can be computed by

$$RCE(x^{k'}, y^{k'}, p) = \frac{\hat{R}(x_f^{k'}, p)}{p y^{k'}}$$
Eq. [23]

One of the problems of DEA is that the reliability of this method heavily depends on the frontier data points. A single outlier could dramatically distort the estimated frontier and efficiency scores. To avoid this problem, we have screened our data before running programs.

Results and Discussion

The linear programs defined above were executed in OnFront (Economic Measurement and Quality in Lund Co., Lund, Sweden). Selected results are reported in Tables 2–7. Table 2 contains the aggregate revenue capacity efficiency measure and its components for each of the 14 grids and for the entire block. Detailed results for individual trees in grids 8 and 9, which were most deficient in macronutrients and micronutrients respectively, are reported

		OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT	OBS	OPT
Row	Column	В	Bo	Zn	Zn°	Mn	Mn°	Cu	Cuº	Fe	Feo
						(mg·kg)				
G	42	0.5	0.3	14.0	13.1	4.0	5.0	17.0	15.0	86.0	96.0
G	43	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
G	44	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
G	45	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
G	46	0.5	0.4	14.0	12.0	4.0	5.6	17.0	12.7	86.0	94.4
G	47	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
G	48	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Ι	41	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Ι	42	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Ι	44	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Ι	45	0.5	0.3	14.0	13.3	4.0	5.1	17.0	15.2	86.0	95.5
Ι	46	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Ι	47	0.5	0.3	14.0	13.1	4.0	5.0	17.0	15.1	86.0	95.8
J	41	0.5	0.3	14.0	13.3	4.0	5.1	17.0	15.2	86.0	95.4
J	43	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
J	45	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
J	46	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
J	47	0.5	0.3	14.0	13.0	4.0	5.0	17.0	15.0	86.0	96.0
Average	e level	0.5	0.3	14.0	13.0	4.0	5.0	17.0	14.9	86.0	95.8

Table 6. Observed (OBS) and optimal (OPT) levels of variable micronutrients B, Zn, Mn, Cu, and Fe (defined as variable factors), for pear trees in grid 9.

Table 7. Aggregate revenue capacity efficiency and ratio of total revenue to total trunk crosssectional area (TCA) for 'OH×F 69', 'OH×F 97', 'OH×F 217', and 'OH×F 333'.

Rootstock	RCE ^z	Rank	Revenue/TCAy	Rank	Revenue ^x	TCAw
OH×F 69	1.713	2	0.0534	2	1577.2	29509.3
OH×F 97	1.727	3	0.0518	3	1801.3	34769.9
OH×F 217	1.751	4	0.0494	4	1850.8	37463.7
OH×F 333	1.465	1	0.0621	1	1687.6	27175

 ${}^{z}RCE$ = aggregate revenue capacity efficiency index when TCA is defined as the only fixed input for all trees in the same rootstocks.

yRatio of total revenue to total TCA for all trees in the same rootstock.

^xTotal revenue of trees in the same rootstock.

"Total trunk cross-sectional area of trees in the same rootstock.

in Tables 3–6. Table 7 compares the economic performances of rootstocks.

For the Hardy block as a whole, the limiting effects of macro- and micronutrients were far less significant than that of environmental constraints (NR1 = 1.068, NR2 = 1.025, and RE = 1.296, Table 2). Furthermore, allocative inefficiency was more severe than technical inefficiency (AE = 1.173 and TE = 1.105, Table 2). This suggests that the major problem of this block was productive inefficiency and specifically, overproduction of low-price pears and underproduction of high-price pears.

Among the 14 grids, grid 1 registered the lowest revenue capacity efficiency index (RCE), closely followed by grids 2 and 14 (Table 2), indicating that trees in these grids economically outperformed those in other grids. By contrast, the RCEs for grids 7 and 8 are far higher than those for other grids (Table 2), suggesting that trees in these grids were suffering relatively severe nutrient deficiency or/and productive inefficiency.

Among the 14 grids, grid 8 was most deficient in macronutrients (NR1 = 1.749, Table 2) and specifically, deficient in NO₃-N, Mg, and S (Table 4). Table 3 shows that even within an individual grid, different trees had different problems. For example, tree G×49 (the tree in row G and column 49) was productively efficient but deficient in macronutrients (RE = TE = AE = 1, NR1

= 1.452, Table 3). This suggests that although this tree achieved the highest revenue in this grid (revenue = \$22.22), its revenue capacity had not been fully realized. Higher revenue could have been achieved if nutrients had been at the optimal levels. Table 4 shows that this tree was deficient in NO₃-N, Mg, and S, and that the optimal levels of these nutrients are 2.4 mg·kg⁻¹, 1.9 meq/100 g, and 8.1 mg·kg⁻¹, in comparison with the observed levels 2.0 mg·kg⁻¹, 1.8 meq/100 g, and 7.0 mg·kg⁻¹, respectively. With almost the same size as tree G×49, tree G×54 only achieved nearly half of its revenue (Table 3). Results show that the major problem for tree G×54 was instead productive inefficiency caused by environmental constraints (RE = 1.939, TE = 1.270, and AE = 1.527, Table 3).

Grid 9 is one of the two grids that were deficient in micronutrients (NR2 = 1.257, Table 2). Specifically, it was deficient in Mn and Fe (Table 6). Similarly, detailed information can be gleaned from Tables 5 and 6 to identify potential problems for individual trees in this grid.

To compare the economic performances of rootstocks, the RCEs associated with TCA were aggregated across trees on the same rootstocks using the same aggregation procedure as before. Table 7 compares the results from our procedure and the commonly adopted aggregation procedure — total revenue divided by

total TCA. Both suggest that 'OH×F 333' was the best rootstock in year 2003, followed by 'OH×F 69' and 'OH×F 97'. 'OH×F 217' had the worst economic performance in this year. To better compare rootstocks in view of their lifespan performances, time series data are required to estimate their temporal productivity changes.

To conclude, we have developed a fruit production model to reveal the hidden information in yield efficiency and its valuebased measure, i.e., the information on nutrient deficiency and productive efficiency. Using the DEA method, we have shown how to utilize this model to identify potential problems for each individual fruit tree and for trees on an aggregate level. Further study should focus on identifying and eliminating the environmental constraints that cause productive inefficiency. Here we have measured productive inefficiency caused by environmental factors but have been silent about how to identify and eliminate these factors owing to lack of data. If adequate data on environmental factors are available, a second-stage regression analysis can be conducted to examine their effects on productive efficiency.

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