A Comparison of Three Mathematical Models of Response to Applied Nitrogen: A Case Study Using Lettuce

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Abstract. Modern fertilization recommendations must optimize crop yield and quality and minimize chances of negative environmental effects due to overfertilization. Data from fertilizer studies can be fitted to several mathematical models to help determine optimum fertilizer rates, but resulting recommendations can vary depending on the model chosen. In this research, lettuce (*Lactuca sativa* L.) was used as a case study vegetable crop to compare models for estimating fertilizer N requirements. Greenhouse studies were conducted with 'South Bay' and 'Sierra' cultivars of crisphead lettuce to measure yield response to applied N. Individual plants were grown in pots and received six rates of N (0.0, 0.2, 0.4, 0.6, 0.8, and 1.0 g/plant) as ammonium nitrate in split applications. Data for plant fresh mass and N uptake were recorded. The logistic model described the data for both cultivars quite well, with correlation coefficients of 0.98 and above. The logistic model was also applied to field data for average head mass of 'South Bay' lettuce following application of N at 0, 56, 112, 168, 224, and 280 kg·ha⁻¹. Logistic, linear-plateau, and quadratic models were compared for the field data. Coefficients for the linear-plateau model were derived from the logistic model. All three models for lettuce production were compared graphically and analytically. The model coefficients were then used to make improved estimates of fertilizer recommendations for field production of lettuce.

Recommendations for fertilization of crops are derived from field and greenhouse studies in which crop yield and quality responses to a range of fertilizer rates are measured. Responses are often modeled to determine optimum fertilizer rate. Today, the relationship of nutrient management to environmental pollution also is an important aspect of any fertilization recommendation. There are many mathematical models for fitting crop response data. The researcher seeks to find a model that describes the data well and aids in defining reasonable fertilization recommendations that result in optimum crop yield and quality without risking overfertilization.

Quadratic models have been very popular for describing crop response to fertilization but tend to overestimate response if the maximum point on the curve is taken as the best fertilization rate. Often, fertilization rates less than the function maximizing rate are statistically similar to the single function maximizing rate (Cerrato and Blackmer, 1990; Hochmuth et al., 1993a).

Models other than quadratic functions have been used to describe crop response to fertilizer. Plateau models, such as linear-plateau and quadratic-plateau (Dahnke and Olson, 1990; Nelson and Anderson, 1977), have been used with agronomic crops (Bullock and Bullock, 1994; Cerrato and Blackmer, 1990; Fageria et al., 1997) and vegetables (Abdul-Baki et al., 1997; Hochmuth et al., 1993a, 1993b; Sanchez et al., 1991), and logistic models with agronomic crops (Overman, 1995; Overman et al., 1990, 1993). More research with vegetable crops to test functions such as the logistic model is needed. Vegetables such as lettuce that require fertilization for optimum yield and quality are ideal candidates for such research.

Lettuce is an important winter vegetable crop that is grown widely throughout the United States, with much of the commercial production in California, Arizona, and southern Florida. Most of Florida's lettuce is produced on Histosol (organic) soils of southern Florida (Hochmuth and Maynard, 1996; Hochmuth et al., 1994). Florida was a major supplier of lettuce (butterhead type) in the early part of the 20th century until the demand for western-grown crisphead lettuce increased (Beckenbach et al., 1941). There is renewed interest in crisphead lettuce production in Florida due to several factors, including loss of portions of the production lands on the Histosols, search for new crops for the sandy

production areas of northern Florida, proximity of northern Florida to eastcoast markets, and development of new production technologies for lettuce, such as polyethylene mulch and drip irrigation (Cantliffe and Karchi, 1992; Hochmuth and Secker, 1994). Because of the high proportion of leaf tissue in lettuce, yields are greatly impacted by N fertilization. Research in Florida with lettuce grown on sandy soils showed that N fertilization requirements were from 120 to 200 kg·ha⁻¹ (Everett, 1980; Hochmuth and Secker, 1994). Sources of N fertilizer did not differ in their effects on crisphead lettuce yield or head quality (Gardner and Pew, 1979). Low levels of N result in small head size and poor yields. Even short periods of N deficiency can have a long-lasting negative effect on lettuce yield (Burns, 1988). Current N recommendation is 135 kg·ha⁻¹ for lettuce grown on sandy soils in Florida (Hochmuth and Maynard, 1996). Yield and N uptake tend to increase linearly with N application rate. At high levels of N, plant yields and N uptake asymptotically approach maximum values.

Decisions concerning optimum rates of fertilization usually involve fitting some type of model to yield data in response to several rates of fertilizer application. Regression analyses have been conducted on numerous data sets for response of agronomic forage crops to applied nutrients (Overman and Evers, 1992; Overman and Wilkinson, 1992; Overman et al., 1990, 1991, 1992, 1993, 1994a, 1994b, 1995). In all these studies, the logistic equation accurately described data for dry-matter yields of forages and corn. In several studies, the extended logistic model also described plant N uptake as well as yield (Overman and Evers, 1992; Overman et al., 1994a, 1994b, 1995). In the latter case, a common N response coefficient, *c*, existed between yield and plant N uptake. As a consequence, yield could be expressed as a hyperbolic function of plant N uptake.

The objective of this study was to demonstrate the utility of the logistic model to describe response of lettuce to applied N. A comparison was made with the linear-plateau and quadratic models for data obtained in both greenhouse and field. Coefficients of the linear-plateau model were obtained as approximations from the logistic model. Both the linear-plateau and quadratic model predicted negative yields at very low N levels, whereas the logistic equation shows asymptotic approach to zero. The general characteristics and a rational basis for the logistic equation have been given by Overman (1995). Output (yield or plant N uptake) remains positive for all applied N, which must be true of the system by definition. Linear-plateau and quadratic models do not meet this constraint.

Materials and Methods

Greenhouse experiment. A greenhouse experiment with lettuce was conducted to verify the suitability of the logistic equation for estimating yield response to applied N. The most healthy and vigorous plants were chosen from

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seedling flats of 'South Bay' (Ithaca head type) and 'Sierra' (French crisphead type) lettuce, two varieties adapted to the growing conditions of the southeastern United States. All plants were started in flats in a growth chamber on 25 Jan. 1994 and received identical nutrient solution fertilization for the first 3 weeks of growth. Selected plants were then transplanted into 15-cm pots containing sterile potting media (70% peat + 30% vermiculite), fertilized with phosphorus (0.3 g P per plant from triple superphosphate) and potassium (0.3 g K per plant from potassium sulfate), and grown until harvest (30 Mar. 1994; 64 d after planting) under optimum environmental conditions in a temperature-controlled greenhouse. Greenhouse temperatures for the month of February ranged from a minimum of 8 to 19 °C to a maximum of 17 to 33 °C. Ammonium nitrate was applied to plants at 39 and 52 d after planting for total N levels of 0.0, 0.2, 0.4, 0.6, 0.8, and 1.0 g/plant. Tap water (200 to 450 mL/plant) was applied from once to several times each week so as to minimize leaching. Treatments were replicated in a randomized block design containing three blocks with 12 plants each. All 36 plants were harvested the same day, weighed, dried at 70 °C for 48 h, weighed again, and ground for chemical analysis. Total Kjeldahl nitrogen (TKN) was determined from 0.2-g samples using standard acid digestion (Jones et al., 1991).

Field experiments. Field experiments were conducted in Spring 1992 and again in 1994 with 'South Bay' lettuce on Arredondo fine sand (loamy siliceous hyperthermic, Grossarenic Paleudults). After soil was prepared by plowing and disking, beds were formed on 2-m centers with 100 kg·ha⁻¹ K incorporated into the soil. Beds were 100 cm wide, with drip irrigation tubes 20 cm on either side of center, and covered with white polyethylene mulch. Lettuce seedlings were transplanted through the mulch on 3 Mar. 1992 and 18 Feb. 1994. Plots were 5 m long and consisted of four rows on 20-cm spacing × 30 cm between plants, for a total of 60 plants per plot (72,000 plants/ha). Ammonium nitrate was injected in eight equal weekly amounts for N levels of 0, 56, 112, 168, 224, and 280 kg·ha⁻¹. Treatments were replicated six times, with irrigation and pest control following recommended cultural practices (Hochmuth and Maynard, 1996). Lettuce heads were harvested on 22 May 1992 and 22 Apr. 1994, and fresh mass of marketable lettuce was recorded.

Model description. Data were analyzed using several models for comparison. The logistic models for yield and plant N uptake are given by

$$Y = A/[1 + \exp(b - cN)] \quad [1]$$

$$N_u = A'/[1 + \exp(b' - cN)] \quad [2]$$

where Y = yield in fresh mass, kg/plant; N_u = nitrogen uptake by lettuce, g/plant; N = nitrogen applied, g/plant or kg·ha⁻¹; A = maximum yield in fresh mass, kg/plant; A' = maximum nitrogen uptake, g/plant; b = intercept parameter for yield; b' = intercept parameter for nitrogen uptake; and c = N response coefficient, plant/g or ha·kg⁻¹. Following Overman

et al. (1994a), Eqs. [1] and [2] can be combined to give the hyperbolic phase relation between yield and plant N uptake

$$Y = Y_m N_u / (K' + N_u) \quad [3]$$

where parameters Y_m and K' are defined in terms of the logistic parameters by

$$Y_m = A/[1 - \exp(b - b')] \quad [4]$$

$$K' = A'[\exp(b' - b) - 1] \quad [5]$$

Note that Y_m represents maximum potential yield and that $N_u = K'$ produces $Y = Y_m/2$, or one-half of maximum potential yield. Calculus techniques show that maximum incremental response to applied N occurs at an application rate $N_{1/2} = b/c$, where $Y = A/2$. This is the point of maximum slope of Y vs. N . Similarly, maximum incremental response of plant N to applied N occurs at $N_{1/2} = b'/c$, with $N_u = A'/2$. The N response coefficient can be redefined as characteristic N given by $N' = 1/c$, which converts units to more familiar g/plant or kg·ha⁻¹.

The linear-plateau model is given by

$$Y_{lp} = B_{lp} + C_{lp}N \text{ for } N < N_x \quad [6]$$

$$Y_{lp} = A_{lp} \text{ for } N > N_x \quad [7]$$

where Y_{lp} = linear-plateau estimate of yield in fresh mass, kg/plant; A_{lp} = plateau or maximum fresh yield, kg/plant; B_{lp} = intercept parameter, kg/plant; C_{lp} = slope parameter, ha/plant; and N_x = N application rate for intersection between Eqs. [6] and [7]. The linear-plateau parameters can be approximated from the logistic parameters as

$$A_{lp} = A \quad [8]$$

$$B_{lp} = A/2(1 - b/2) = A/4N'(2N' - N_{1/2}) \quad [9]$$

$$C_{lp} = A/c/4 = A/4N' \quad [10]$$

This occurs because the logistic model approximates a straight line in the midrange of response. It follows that the intersection of the linear and plateau portions occurs at

$$N_x = (A_{lp} - B_{lp})/C_{lp} = (b + 2)/c = N_{1/2} + 2N' \quad [11]$$

The quadratic model can be written as

Fig. 1. Fresh mass and N uptake of lettuce in response to applied N in the greenhouse at Gainesville, Fla. Curves drawn from Eqs. [1] and [2] with logistic parameters from Table 1.

Table 1. Logistic parameters for greenhouse lettuce at Gainesville, Fla.

Component	Cultivar	A g/plant	b	c plants/g	$N_{1/2}$ g/plant	N' g/plant
Fresh mass	South Bay	440	1.50	5.45	0.275	0.183
	Sierra	470	1.50	5.45	0.275	0.183
N uptake	South Bay	0.625	2.13	5.45	0.391	0.183
	Sierra	0.780	2.13	5.45	0.391	0.183

$$Y_q = A_q + B_q N - C_q N^2 \quad [12]$$

where Y_q = quadratic estimate of yield in fresh mass, kg/plant; A_q = intercept parameter, kg/plant; B_q = linear response coefficient, ha/plant; and C_q = quadratic response coefficient, $\text{ha}^2 \cdot \text{kg}^{-1}$ per plant. Peak production can be estimated from the maximum where the derivative, $dY_q/dN = 0$, which occurs at

$$N_{\text{peak}} = B_q / 2C_q \quad [13]$$

and gives peak production of

$$\begin{aligned} Y_{\text{peak}} &= A_q + B_q^2 / 4C_q \\ &= A_q + B_q / 2N_{\text{peak}} \end{aligned} \quad [14]$$

Fertilization rates of N_{peak} may not be optimal for production because of diminishing returns obtained as N approaches N_{peak} . Therefore, optimum applied N rates would tend to be below N_{peak} (i.e., $N_{\text{opt}} < N_{\text{peak}}$).

Results and Discussion

Greenhouse experiment. Response of lettuce to applied N is shown in Fig. 1. The curves are drawn from Eqs. [1] and [2] with parameters listed in Table 1. Data at $N = 0$ were omitted from the regression analysis because of severe nutrient deficiency. The N response coefficient, c , was assumed common for all curves, while b was assumed common for the two cultivars, as was b' . Logistic models show monotone increasing response in agreement with the data. In the phase relation (Fig. 2), the curves are drawn from Eqs. [3] through [5] with the parameters $Y_m = 0.94$ kg/plant and $K' = 0.712$ g/plant for 'South Bay', and $Y_m = 1.00$ kg/plant and $K' = 0.889$ g/plant for 'Sierra'. The hyperbolic relationship agrees with the data rather well. We conclude that the extended logistic model described yield and plant N response to applied N quite well. According to Fig. 2, plant yield is ultimately limited by some factor other than N uptake (such as solar radiation, atmospheric carbon dioxide concentration, etc.).

Field experiments. Response of field lettuce to applied N is shown in Fig. 3. Logistic, linear-plateau, and quadratic models were fitted to the data with parameters listed in Table 2. The logistic model provides a reasonable basis for the linear-plateau model (Fig. 3). The intersection point can be calculated from Eq. [11], and peak N values for the quadratic model were calculated from Eq. [13]. A summary of critical values of model parameters is listed in Table 3. At $N = N_{1/2}$ yield is 50% of the plateau, whereas at $N = N_x$ yield is 88% of plateau. For $N = N_{\text{peak}}$ yields are well out on the plateau, beyond the region of significant response to applied N .

From these results, the logistic model apparently provides an adequate description of both greenhouse and field results for response of lettuce to applied N .

Summary. From analysis of the field studies, N_x appears to give the most reasonable level for a nitrogen fertilizer recommendation, viz., $\approx 70 \text{ kg} \cdot \text{ha}^{-1}$ for these conditions. This is considerably below the current Florida recommendation of $135 \text{ kg} \cdot \text{ha}^{-1}$ (Hochmuth and Clark,

Fig. 2. Dependence of fresh mass on plant N uptake for greenhouse lettuce at Gainesville, Fla. Curves drawn from Eq. [3] with parameters calculated by Eqs. [4] and [5] using logistic parameters from Table 1.

Fig. 3. Comparison of logistic, linear-plateau, and quadratic models for response of field-grown lettuce to N application at Gainesville, Fla. Model values calculated with parameters from Table 2.

Table 2. Model parameters for field lettuce at Gainesville, Fla.

Model	Parameter	Value	
		1992	1994
Logistic	A, kg/plant	0.510	0.729
	b	0.70	1.09
	c, ha·kg ⁻¹	0.0346	0.0459
Linear-plateau	A _{lp} , kg/plant	0.510	0.729
	B _{lp} , kg/plant	0.166	0.166
	C _{lp} , ha/plant	0.00441	0.00837
Quadratic	A _q , kg/plant	0.186	0.233
	B _q , ha/plant	0.00381	0.00573
	C _q , ha ² ·kg ⁻¹ /plant	0.0000101	0.0000147

Table 3. Critical N values (kg·ha⁻¹) for the models for field-grown lettuce at Gainesville, Fla.

Year	N _{1/2}	N _x	N _{peak}
1992	20	78	189
1994	24	67	195

1991). Injection of N through the drip irrigation system may lead to more efficient use of N by the crop, since this reduces the risk of N leaching at any given period in the season. This is an important consideration given current emphasis on groundwater contamination by nitrates from crop fertilization and on environmental accountability.

The logistic model offers a useful tool for evaluation of lettuce response to applied N. Parameters A, b, and c in Eq. [1] can be estimated from data by nonlinear regression. One can also use the following simple alternative procedure. Parameter A (the plateau) can be estimated by visual inspection of the data for yield vs. applied N (such as Fig. 1 or Fig. 3). Then parameter b follows from

$$b = \ln(A/Y_0 - 1) \quad [15]$$

where Y₀ = estimated intercept yield at N = 0. Finally, parameter c is calculated from

$$c = b/N_{1/2} \quad [16]$$

where N_{1/2} is estimated as the value of N corresponding to y = A/2 (50% of the plateau) on the graph of yield response to applied N. With parameters b and c in hand, N_x can then be estimated from Eq. [11]. Estimates of yield at given applied N levels are easily made with Eq. [1] using a calculator with an equation writer.

We encourage further studies on the application of the logistic model to response of horticultural crops to applied N. The model contains the right characteristics to describe field data and is relatively simple to use in practice. As noted in the literature section, this model has been successfully applied to numerous studies with forage crops.

Literature Cited

- Abdul-Baki, A.A., J.R. Teasdale, and R.F. Korkak. 1997. Nitrogen requirements of fresh-market tomatoes on hairy vetch and black polyethylene mulch. *HortScience* 32:217-221.
- Beckenbach, J.R., F.S. Jamison, R.W. Ruprecht, and F.S. Andrews. 1941. Crisp-head lettuce in Florida: A preliminary report. *Florida Agr. Expt. Sta. Bul.* 365.
- Bullock, D.G. and D.S. Bullock. 1994. Quadratic and quadratic-plateau models for predicting optimum nitrogen rate of corn: A comparison. *Agron. J.* 86:191-195.
- Burns, I.G. 1988. Effect of interruptions in N, P, or K supply on the growth and development of lettuce. *J. Plant Nutr.* 11:1627-1634.
- Cantliffe, D.J. and Z. Karchi. 1992. Performance of crisphead lettuce cultivars on polyethylene-mulched drip-irrigated sandy soils in Florida. *Proc. Fla. State Hort. Soc.* 105:340-342.
- Cerrato, M.E. and A.M. Blackmer. 1990. Comparison of models for describing corn yield response to nitrogen fertilizer. *Agron. J.* 82:138-143.
- Dahnke, W.C. and R.A. Olson. 1990. Soil test correlation, calibration, and recommendation, p. 45-71. In: R.L. Westerman (ed.). *Soil testing and plant analysis*, 3rd ed. Soil Sci. Soc. Amer., Madison, Wis.
- Everett, P. 1980. Influence of fertilizer rates and plastic mulch on the production of two cultivars of crisphead lettuce. *Proc. Fla. State Hort. Soc.* 93:243-245.
- Fageria, N.K., A.B. Santos, and C. Baligar. 1997. Phosphorus soil test calibration for lowland rice on an Inceptisol. *Agron. J.* 737-742.
- Gardner, B.R. and W.D. Pew. 1979. Comparison of various nitrogen sources for the fertilization of winter-grown head lettuce. *J. Amer. Soc. Hort. Sci.* 104:534-536.
- Hochmuth, G.J. and G.A. Clark. 1991. Fertilizer application and management for micro (or drip) irrigated vegetables in Florida. *Florida Coop. Ext. Serv. Special Series Rpt. SS-VEC-45*.
- Hochmuth, G.J., E.A. Hanlon, and J. Cornell. 1993a. Watermelon phosphorus requirements in soils with low Mehlich-1 extractable phosphorus. *HortScience* 28:630-632.

- Hochmuth, G.J., E.A. Hanlon, R. Nagata, G. Snyder, and T. Schueneman. 1994. Fertilization recommendations for crisphead lettuce grown on organic soils in Florida. *Florida Coop. Ext. Serv. Cir. SP-153*.
- Hochmuth, G.J., R.C. Hochmuth, M.E. Donley, and E.A. Hanlon. 1993b. Eggplant yield response to potassium fertilization on sandy soil. *HortScience* 28:1002-1005.
- Hochmuth, G.J. and D.N. Maynard (eds.). 1996. *Commercial vegetable production guide for Florida*. Florida Coop. Ext. Serv. Circ. SP-170.
- Hochmuth, G.J. and I. Secker. 1994. Nitrogen requirements of crisphead lettuce grown with drip irrigation on polyethylene-mulched beds. *Proc. Amer. Soc. Plasticult.* 25:96-100.
- Jones, J.B., Jr., B. Wolf, and H.A. Mills. 1991. *Plant analysis handbook: A practical sampling, preparation, analysis, and interpretation guide*. Micro-Macro Publ., Athens, Ga.
- Nelson, L.A. and R.L. Anderson. 1997. Partitioning of soil-test response probability, p. 19-38. In: T.R. Peck, J.T. Cope, and D.A. Whitney (eds.). *Soil testing: Correlation and interpreting the analytical results*. Amer. Soc. Agron., Madison, Wis. Publ. 29.
- Overman, A.R. 1995. Rational basis for the logistic model for forage grasses. *J. Plant Nutr.* 18:995-1012.
- Overman, A.R., A. Dagan, F.G. Martin, and S.R. Wilkinson. 1991. A nitrogen-phosphorus-potassium model for forage yield of bermudagrass. *Agron. J.* 83:254-258.
- Overman, A.R. and G.W. Evers. 1992. Estimation of yield and nitrogen removal by bermudagrass and bahiagrass. *Trans. Amer. Soc. Agr. Eng.* 35:207-210.
- Overman, A.R., G.W. Evers, and S.R. Wilkinson. 1995. Coupling of dry matter and nutrient accumulation in forage grass. *J. Plant Nutr.* 18:2629-2642.
- Overman, A.R., F.G. Martin, and S.R. Wilkinson. 1990. A logistic equation for yield response of forage grass to nitrogen. *Commun. Soil Sci. Plant Anal.* 21:595-609.
- Overman, A.R., M.A. Sanderson, and R.M. Jones. 1993. Logistic response of bermudagrass and bunchgrass cultivars to applied nitrogen. *Agron. J.* 85:541-545.
- Overman, A.R. and S.R. Wilkinson. 1992. Model evaluation for perennial grasses in the southern United States. *Agron. J.* 84:523-529.
- Overman, A.R., S.R. Wilkinson, and G.W. Evers. 1992. Yield response of bermudagrass and bahiagrass to applied nitrogen and overseeded clover. *Agron. J.* 84:998-1001.
- Overman, A.R., S.R. Wilkinson, and D.M. Wilson. 1994a. An extended model of forage grass response to applied nitrogen. *Agron. J.* 86:617-620.
- Overman, A.R., D.M. Wilson, and E.J. Kamprath. 1994b. Estimation of yield and nitrogen removal by corn. *Agron. J.* 86:1012-1016.
- Sanchez, C.A., M. Lockhart, and P.S. Porter. 1991. Response of radish to phosphorus and potassium fertilization on Histosols. *HortScience* 26:30-32.